I. Non-relativistic limit of scalar field theory. | Examen SQFT January 2015 : correct° |

2) $$\frac{\partial \mathcal{L}}{\partial \phi} = -m^2 \phi^* - 2\lambda (\phi^*)^2 \phi$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = \partial^\mu \phi^*$$

$$EL \to \partial_\mu \partial^\mu \phi^* = -m^2 \phi^* - 2\lambda |\phi|^2 \phi^*$$

$$\left[\Box + m^2 + 2\lambda |\phi|^2\right] \phi = 0$$

$$\Pi_\phi = \frac{\partial \mathcal{L}}{\partial \dot\phi} = \dot\phi^*$$

3) $$\Box = \partial_t^2 - \vec\nabla^2$$

$$\partial_t \phi = \frac{1}{\sqrt{2m}} e^{-imt}(-im\varphi + \dot\varphi)$$

$$\partial_t^2 \phi = \frac{1}{\sqrt{2m}} e^{-imt} \times -im(-im\varphi + \dot\varphi) + \frac{1}{\sqrt{2m}} e^{-imt}(-im\dot\varphi + \ddot\varphi) \quad - \vec\nabla^2 \varphi$$

$$\Big[\cancel{-m^2\varphi} - im\dot\varphi - im\dot\varphi + \underbrace{\ddot\varphi}_{\text{negligeable}} \cancel{+m^2\varphi} + \frac{2\lambda}{2m}|\varphi|^2\varphi\Big] = 0$$

$$-2im\dot\varphi - \Delta\varphi + \frac{\lambda}{m}|\varphi|^2 \simeq 0$$

$$i\dot\varphi \simeq -\frac{1}{2m}\Delta\varphi + \underbrace{\frac{\lambda}{2m^2}}_{\equiv g}|\varphi|^2$$

4) $$\phi = \frac{1}{\sqrt{2m}} e^{-imt} \varphi$$

$$\partial_0 \phi = \frac{1}{\sqrt{2m}} e^{-imt}(-im\varphi + \dot\varphi)$$

$$\partial_i \phi = \frac{1}{\sqrt{2m}} e^{-imt} \partial_i \varphi$$

$$\mathcal{L} = (\partial_0\phi^*)(\partial_0\phi) - (\partial_i\phi^*)(\partial_i\phi) - m^2\phi^*\phi - \lambda(\phi^*\phi)^2$$

$$= \frac{1}{2m}\underbrace{(+im\varphi^* + \dot\varphi^*)(-im\varphi + \dot\varphi)}_{} - \frac{1}{2m}(\partial_i\varphi^*)(\partial_i\varphi) \cancel{- \frac{m^2}{2m}\varphi^*\varphi} - \frac{\lambda}{(2m)^2}(\varphi^*\varphi)^2$$

$$\cancel{+m^2(\varphi^*\varphi)} + im\varphi^*\dot\varphi - im\varphi\dot\varphi^* + \underbrace{\dot\varphi^*\dot\varphi}_{\text{negligeable}}$$

$$\overset{IPP}{\simeq} \underbrace{\frac{i}{2}\varphi^*\dot\varphi + i\frac{\varphi^*\dot\varphi}{2}}_{i\varphi^*\dot\varphi} - \frac{1}{2m}\vec\nabla\varphi^* \cdot \vec\nabla\varphi - \frac{\lambda}{2\times 2m^2}(\varphi^*\varphi)^2$$

$$\frac{\lambda}{2m^2} \equiv g$$

$$\frac{\partial \mathcal{L}}{\partial \varphi^*} = i\dot\varphi - \frac{g}{2}\varphi^2 2\varphi^*$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi^*)} = \begin{cases} 0 & \text{si } \mu = 0 \\ -\frac{1}{2m}\partial_i\varphi & \text{si } \mu = i \end{cases}$$

$$EL \to -\frac{1}{2m}\vec\nabla^2\varphi = i\dot\varphi - g|\varphi|^2\varphi$$

$$\to i\partial_t\varphi = -\frac{1}{2m}\Delta\varphi + g|\varphi|^2\varphi \quad \text{ok}$$

5) $$\Pi_\varphi = \frac{\partial \mathcal{L}}{\partial \dot\varphi} = i\varphi^*$$

6) $$\mathcal{H} = \Pi_\varphi\dot\varphi - \mathcal{L} = i\varphi^*\dot\varphi - i\varphi^*\dot\varphi + \frac{1}{2m}\vec\nabla\varphi^* \cdot \vec\nabla\varphi + \frac{g}{2}|\varphi|^4$$

$$H = \int d^3x \left[\frac{1}{2m}\vec\nabla\varphi^* \cdot \vec\nabla\varphi + \frac{g}{2}|\varphi|^4\right] = \int d^3x \left\{\frac{1}{2mi}\vec\nabla\Pi_\varphi \cdot \vec\nabla\varphi \pm \frac{g}{2}\Pi_\varphi^2\varphi^2\right\}$$

7) $J^\mu = i\phi^*\partial^\mu\phi - i\phi\partial^\mu\phi^*$

$J^0 = i\phi^*\partial_t\phi - i\phi\partial_t\phi^*$

$\simeq \frac{i}{2m}\varphi^*(-im\varphi + \dot\varphi) + cc$

$\simeq \frac{\varphi^*\varphi}{2} + cc = \varphi^*\varphi$

$\vec J = -i\phi^*\vec\nabla\phi + cc = -\frac{i}{2m}\varphi^*\vec\nabla\varphi + cc. = \frac{1}{2}\left(\varphi^* \times \frac{-i\vec\nabla}{m}\varphi + \varphi \times \frac{i\vec\nabla}{m}\varphi^*\right)$

$\partial_\mu J^\mu = 0 = \partial_0 J^0 + \partial_i J^i = \partial_t J^0 + \vec\nabla\cdot\vec J$

ie. $\partial_t(\varphi^*\varphi) + \vec\nabla\cdot\left(-\frac{i}{2m}\varphi^*\vec\nabla\varphi + cc.\right) = 0$

$J^0 = \varphi^*\varphi$ density of particles

$Q = \int d^3x\, \varphi^*\varphi$ number of particles.

in the relativistic case $Q =$ nb of particles − nb of antiparticles = nb of electric charges.

8) no question $\mathcal{L} = i\varphi^*\dot\varphi - \frac{1}{2m}\vec\nabla\varphi^*\cdot\vec\nabla\varphi + \mu|\varphi|^2 - \frac{g}{2}|\varphi|^4$

9) $\mathcal{H} = \Pi_\varphi\dot\varphi - \mathcal{L} = \frac{1}{2m}\vec\nabla\varphi^*\cdot\vec\nabla\varphi \underbrace{-\mu|\varphi|^2 + \frac{g}{2}|\varphi|^4}_{V(\varphi^*\varphi)}$

$\mu<0$:  $\varphi_{eq} = 0$ stable unique $\quad V(|\varphi|) = -\mu|\varphi|^2 + \frac{g}{2}|\varphi|^4 \quad V'(|\varphi|) = -2\mu|\varphi| + 2g|\varphi|^3$
$0 = |\varphi|$

$V''(|\varphi|) = -2\mu + 4g|\varphi|^2 \overset{=0}{=} -2\mu > 0$

$\mu>0$: 

$\varphi_{eq} = 0$ unstable

$V'(|\varphi|) = 2|\varphi|(-\mu + g|\varphi|^2) = 0 \rightarrow |\varphi| = \sqrt{\frac{\mu}{g}}$

$V''(|\varphi|) = -2\mu + 4g|\varphi|^2 = -2\mu + 4g\times\frac{\mu}{g} = 2\mu > 0$ stable

highly degenerate $\varphi_{eq} = \sqrt{\frac{\mu}{g}}\, e^{i\theta} \quad \forall\theta$.

$\mu = 0$: phase transit°, critical point.

10) $\varphi = \sqrt{n}\, e^{i\theta} \qquad \dot\varphi = \frac{\dot n}{2\sqrt n}e^{i\theta} + i\dot\theta\sqrt n\, e^{i\theta} \qquad \nabla\varphi = \frac{\nabla n}{2\sqrt n}e^{i\theta} + \sqrt n\, e^{i\theta}\, i\nabla\theta \qquad |\varphi|^2 = n,\ |\varphi|^4 = n^2$

$\mathcal{L} = i\sqrt n\left(\frac{\dot n}{2\sqrt n} + i\dot\theta\sqrt n\right) - \frac{1}{2m}\left(\frac{\nabla n}{2\sqrt n} - i\sqrt n\,\nabla\theta\right)\left(\frac{\nabla n}{2\sqrt n} + i\nabla\theta\sqrt n\right) + \mu n - \frac{g}{2}n^2$

$= i\frac{\dot n}{2} - \dot\theta n - \frac{1}{2m}\left[\frac{\nabla n\cdot\nabla n}{4n} + n\,\nabla\theta\cdot\nabla\theta\right] + \mu n - \frac{gn^2}{2} = \frac{i}{2}\dot n + (\mu - \dot\theta)n - \frac{n}{2m}(\nabla\theta)^2 - \frac{(\nabla n)^2}{8mn} - \frac{gn^2}{2}$

total derivative

$\Pi_\theta = \frac{\partial\mathcal{L}}{\partial\dot\theta} = -n \qquad [\theta(\vec x,t), -n(\vec x',t)] = i\delta(\vec x - \vec x') \rightarrow [N(t), \theta(\vec x,t)] = i$

11) $n = \bar n + \rho,\ |\rho| \ll 1,\ |\theta| \ll 1 \qquad \varphi_0 = \sqrt{\frac{\mu}{g}}\, e^{i0} \qquad \bar n = \mu/g$

$\frac{g\mu^2}{2g^2}$

$\mathcal{L} = (\mu - \dot\theta)(\bar n + \rho) - \frac{\bar n}{2m}(\nabla\theta)^2 - \frac{(\nabla\rho)^2}{8m\bar n} - \frac{g}{2}(\bar n^2 + 2\bar n\rho + \rho^2) + \dots$

on enlève les ctes et les dérivées totales

$= \frac{\mu^2}{g} + \cancel{\mu\rho} - \dot\theta\bar n - \rho\dot\theta - \frac{\bar n}{2m}(\nabla\theta)^2 - (\nabla\rho)^2/8m\bar n - \cancel{\frac{\mu^2}{2g}} - \cancel{\mu\rho} - \frac{g\rho^2}{2} + \dots \simeq -\rho\dot\theta - \frac{\bar n}{2m}(\nabla\theta)^2 - \frac{(\nabla\rho)^2}{8m\bar n} - \frac{g\rho^2}{2}$

12) $\frac{\partial \mathcal{L}}{\partial \theta} = 0 \qquad \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = -\rho \qquad \frac{\partial \mathcal{L}}{\partial (\nabla\theta)} = -\frac{\bar{n}}{m}\nabla\theta$

EL $\rightarrow \quad \underline{-\dot{\rho} - \frac{\bar{n}}{m}\nabla^2\theta = 0 \quad (1)} \qquad \frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\Big(\underbrace{\frac{\bar{n}}{m}\vec{\nabla}\theta}_{\sim \vec{v}}\Big) = 0$

$\frac{\partial \mathcal{L}}{\partial \rho} = -g\rho - \dot{\theta} \quad \frac{\partial \mathcal{L}}{\partial \dot{\rho}} = 0 \quad \frac{\partial \mathcal{L}}{\partial (\nabla\rho)} = -\frac{\nabla\rho}{4m\bar{n}}$

EL $\rightarrow \quad \underline{0 - \frac{\nabla^2\rho}{4m\bar{n}} + 0 = -g\rho - \dot{\theta} \quad (2)} \qquad m\frac{\partial}{\partial t}\left(\frac{\nabla\vec{\theta}}{m}\right) = -g\vec{\nabla}\rho + \frac{\vec{\nabla}}{4m\bar{n}}\left(\vec{\nabla}^2\rho\right)$

$-\ddot{\rho} - \frac{\bar{n}}{m}\nabla^2\dot{\theta} = 0 = -\ddot{\rho} - \frac{\bar{n}}{m}\nabla^2\left(-g\rho + \frac{\nabla^2\rho}{4m\bar{n}}\right)$ i.e. $\ddot{\rho} - \frac{g\bar{n}}{m}\nabla^2\rho + \frac{\nabla^4\rho}{(2m)^2} = 0$

13) $\rho(x) = \rho\, e^{-ik\cdot x} \rightarrow (-i\omega)^2 - \frac{g\bar{n}}{m}(i\vec{k})^2 + \frac{(i\vec{k})^4}{(2m)^2} = 0$

$-\omega^2 + \frac{g\bar{n}}{m}\vec{k}^2 + \left(\frac{\vec{k}^2}{2m}\right)^2 = 0 \rightarrow \omega = \sqrt{\frac{\vec{k}^2}{2m}\left(2g\bar{n} + \frac{\vec{k}^2}{2m}\right)} \simeq \sqrt{\frac{g\bar{n}}{m}}|\vec{k}|$

sound waves

14) $v_s = \sqrt{\frac{g\bar{n}}{m}} \sim$ mm/s

15) Goldstone mode = sound wave, SSB of $U(1)$
single mode, single coupled phase-amplitude mode
not two modes

II.

Maxwell

1) $\frac{\partial \mathcal{L}_M}{\partial A_\nu} = -J^\nu \qquad \frac{\partial \mathcal{L}_M}{\partial(\partial_\mu A_\nu)} = -F^{\mu\nu} \qquad EL \rightarrow \partial_\mu F^{\mu\nu} = J^\nu$

$\left(\Box A^\nu - \partial^\nu(\partial\cdot A) = J^\nu\right)$

$\partial_\nu J^\nu = \partial_\nu \partial_\mu F^{\mu\nu} = 0$

2) $\mathcal{L}_M \longrightarrow -\frac{1}{4}F^2 - (A_\mu + \partial_\mu\theta) J^\mu = \mathcal{L}_M - \partial_\mu\theta . J^\mu$

$S_M \rightarrow S_M - \int d^4x\, J^\mu \partial_\mu \theta \underset{IPP}{=} S_M + \int d^4x\, \theta \underbrace{\partial_\mu J^\mu}_{=0} = S_M$

3) $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

$\vec{E} = \begin{cases} E_x = -\partial_t A_x - \partial_x A_0 \\ E_y = -\partial_t A_y - \partial_y A_0 \end{cases} \qquad B = \partial_x A_y - \partial_y A_x$

$F^{01} = \partial^0 A^1 - \partial^1 A^0 = \partial_0 A^1 + \partial_1 A_0 = \partial_t A_x + \partial_x A_0 = -E_x$

$F^{02} = \partial^0 A^2 - \partial^2 A^0 = \partial_0 A^2 + \partial_2 A_0 = \partial_t A_y + \partial_y A_0 = -E_y$

$F^{12} = \partial^1 A^2 - \partial^2 A^1 = -\partial_1 A^2 + \partial_2 A^1 = \partial_y A_x - \partial_x A_y = -B$

$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y \\ E_x & 0 & -B \\ E_y & B & 0 \end{pmatrix}$

$x^\mu = (x^0, x^1, x^2) \xrightarrow{P} (x^0, -x^1, x^2)$ i.e. $x^0 \rightarrow x^0$ and $\vec{x} \rightarrow (-x^1, x^2)$

$p^\mu = i\partial^\mu = i(\partial^0, \partial^1, \partial^2) \xrightarrow{P} i(\partial^0, -\partial^1, \partial^2)$ i.e. $p^0 \rightarrow p^0$ et $\vec{p} \rightarrow (-p_x, p_y)$

$A^\mu = (A^0, A^1, A^2) \xrightarrow{P} (A^0, -A^1, A^2)$

$B = \partial_2 A^1 - \partial_1 A^2 \xrightarrow{P} \partial_2(-A^1) - (-\partial_1)A^2 = -B$: pseudoscalar

$E_x = -\partial_0 A^1 - \partial_1 A_0 \xrightarrow{P} -\partial_0(-A^1) - (-\partial_1) A_0 = -E_x$

$E_y = -\partial_0 A^2 - \partial_2 A_0 \xrightarrow{P} -\partial_0 A^2 - \partial_2 A_0 = E_y$

$\}$ true vector

$F_{\mu\nu} = \begin{pmatrix} 0 & +E_x & +E_y \\ -E_x & 0 & -B \\ -E_y & B & 0 \end{pmatrix}$

4) $\partial_\mu F^{\mu\nu} = J^\nu$ et $\partial_\mu \tilde{F}^\mu = 0$ i.e. $\partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} + \partial_\rho F_{\mu\nu} = 0$ on avec $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

$J^0 = \partial_i F^{i0} = -\partial_i F^{0i} = -\partial_1 F^{01} - \partial_2 F^{02} = \partial_x E_x + \partial_y E_y = \rho$ i.e. $\text{div}\, \vec{E} = \rho$

$J^1 = \partial_\mu F^{\mu 1} = \partial_0 F^{01} + \partial_2 F^{21} = -\partial_t E_x + \partial_y B$ i.e. $\partial_y B - \partial_t E_x = J_x$

$J^2 = \partial_0 F^{02} + \partial_1 F^{12} = -\partial_t E_y - \partial_x B$ i.e. $-\partial_x B - \partial_t E_y = J_y$

$\}$ i.e. $\vec{\nabla}\times\vec{B} - \partial_t \vec{E} = \vec{J}$

$\begin{pmatrix}\partial_x \\ \partial_y \\ 0\end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ B\end{pmatrix} - \partial_t \begin{pmatrix} E_x \\ E_y \\ 0\end{pmatrix} = \begin{pmatrix} J_x \\ J_y \\ 0\end{pmatrix}$

$\partial_0 F_{12} + \partial_1 F_{20} + \partial_2 F_{01} = 0$ i.e. $\partial_t(-B) + \partial_x(-E_y) + \partial_y E_x = 0$ i.e. $(\vec{\nabla}\times\vec{E})_z + \partial_t B = 0$

~~$\partial_0 F_{21} + \partial_2 F_{10} + \partial_1 F_{02} = 0$ i.e. $\partial_t B + \partial_y(-E_x) + \partial_x E_y = 0$~~

$\vec{\nabla}\times\vec{E} + \partial_t \vec{B} = 0$

il n'y a pas $\vec{\nabla}\cdot\vec{B} = 0$ car $\partial_x 0 + \partial_y 0 + \partial_z B(x,y,t,\cancel{z}) = 0$

2+1: 4 éqts

3+1: 8 éqts

5) $\Box A^\nu - \partial^\nu(\partial_\mu A^\mu) = J^\nu = 0$

radiat° gauge: $A^0 = 0$ & $\vec{\nabla}\cdot\vec{A} = 0 \Rightarrow \partial_\mu A^\mu = 0 \Rightarrow \Box A^\nu = 0 \Rightarrow \omega = \pm|\vec{k}|$

$\vec{A}(x) = \underline{\vec{A}} e^{-ik\cdot x} \qquad \vec{\nabla}\cdot\vec{A} = i\vec{k}\cdot\underline{\vec{A}} e^{-ik\cdot x} = 0 \Rightarrow \underline{\vec{A}} \perp \vec{k} \Rightarrow$ 1 mode

Proca

$\rightarrow \Box A^\nu - \partial^\nu(\partial\cdot A)$

7) $\partial_\mu F^{\mu\nu} + m_A^2 A^\nu = 0$

$\underbrace{\partial_\nu \partial_\mu F^{\mu\nu}}_{=0} + m_A^2 \partial_\nu A^\nu = 0 \underset{m_A \neq 0}{\Rightarrow} \partial_\nu A^\nu = 0$

$\} \quad (\Box + m_A^2) A^\nu = 0$

massive vector field but not a gauge field.

8) $\omega = \pm\sqrt{\vec{k}^2+m^2}$ 2 modes ($\partial_\mu A^\mu = 0$ removes one mode from the three)

Chern-Simons

9) $\mathcal{L}_{CS} \to \frac{K}{2}\epsilon^{\mu\nu\rho}(A_\mu+\partial_\mu\theta)\partial_\nu(A_\rho+\cancel{\partial_\rho\theta}) - (A_\mu+\partial_\mu\theta)J^\mu$

$= \mathcal{L}_{CS} + \frac{K}{2}\epsilon^{\mu\nu\rho}\underbrace{\partial_\mu\theta . \partial_\nu A_\rho}_{} - \partial_\mu\theta . J^\mu$

$\to \partial_\mu(\theta\partial_\nu A_\rho)$ because $\epsilon^{\mu\nu\rho}\theta\partial_\mu\partial_\nu A_\rho = 0.$

$$S_{CS} \to S_{CS} + \frac{K}{2}\epsilon^{\mu\nu\rho}\underbrace{\int d^3x\, \partial_\mu(\theta\partial_\nu A_\rho)}_{\substack{\text{total derivative}\\ \to 0}} - \underbrace{\int d^3x\, \partial_\mu\theta . J^\mu}_{\overset{=}{\scriptscriptstyle IPP} -\int d^3x\, \theta \underbrace{\partial_\mu J^\mu}_{=0} = 0} = S_{CS}$$

if boundary terms can be dropped.

10) $\frac{\partial\mathcal{L}_{CS}}{\partial A_\nu} = -J^\nu + \frac{K}{2}\epsilon^{\nu\mu\rho}\partial_\mu A_\rho \qquad \frac{\partial\mathcal{L}_{CS}}{\partial(\partial_\mu A_\nu)} = \frac{K}{2}\epsilon^{\rho\mu\nu}A_\rho$

EL $\to -J^\nu + \frac{K}{2}\epsilon^{\nu\mu\rho}\partial_\mu A_\rho = \frac{K}{2}\epsilon^{\rho\mu\nu}\partial_\mu A_\rho \to J^\nu = K\epsilon^{\nu\mu\rho}\partial_\mu A_\rho$ $= \frac{K}{2}\epsilon^{\nu\mu\rho}(\partial_\mu A_\rho - \partial_\rho A_\mu)$ $= K\epsilon^{\nu\mu\rho}\frac{F_{\mu\rho}}{2} = K\tilde{F}^\nu$

$J^\nu = 0 \to \tilde{F}^\nu = 0 \to F_{\mu\rho} = 0 \to A_\mu = \partial_\mu\theta$ is a pure gauge.

11) $\Pi_{A_\nu} = \Pi^\nu = \frac{\partial\mathcal{L}}{\partial\dot{A}_\nu} = \frac{K}{2}\epsilon^{\mu 0\nu}A_\mu = -\frac{K}{2}\epsilon^{0ij}A_j \qquad \Pi^0 = 0 \quad \Pi^i = -\frac{K}{2}\epsilon^{ij}A_j$

$\mathcal{H} = \Pi^\nu\dot{A}_\nu - \mathcal{L} = \frac{K}{2}\epsilon^{\mu 0\nu}A_\mu\partial_0 A_\nu - \frac{K}{2}\epsilon^{\mu\rho\gamma}A_\mu\partial_\rho A_\gamma$

$= -\frac{K}{2}\epsilon^{\mu i\nu}A_\mu\partial_i A_\nu = 0$ as ~~A~~ μ or $\nu = 0$ and $A_0 = 0$

$\to H = 0.$

MCS

12) yes

13) $-\partial_\mu F^{\mu\nu} + \frac{K}{2}\epsilon^{\rho\mu\nu}\partial_\mu A_\rho = \frac{K}{2}\epsilon^{\nu\mu\rho}\partial_\mu A_\rho \to \partial_\mu F^{\mu\nu} + K\epsilon^{\nu\mu\rho}\partial_\mu A_\rho = 0$

$\partial_\mu F^{\mu\nu} + \frac{K}{2}\epsilon^{\nu\mu\rho}F_{\mu\rho} = 0 = \partial_\mu F^{\mu\nu} + K\tilde{F}^\nu$

14) $F^{\mu\nu} \overset{?}{=} \epsilon^{\mu\nu\rho}\tilde{F}_\rho = \epsilon^{\mu\nu\rho}\frac{1}{2}\epsilon_{\rho\alpha\beta}F^{\alpha\beta} = \frac{1}{2}\epsilon^{\rho\mu\nu}\epsilon_{\rho\alpha\beta}F^{\alpha\beta} = \frac{1}{2}(\delta^\mu_\alpha\delta^\nu_\beta - \delta^\mu_\beta\delta^\nu_\alpha)F^{\alpha\beta} = \frac{1}{2}(F^{\mu\nu} - F^{\nu\mu}) = F^{\mu\nu}$

$\partial_\mu\tilde{F}^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho}\partial_\mu F_{\nu\rho} = \frac{1}{2}\epsilon^{\mu\nu\rho}(\partial_\mu\partial_\nu A_\rho - \partial_\mu\partial_\rho A_\nu) = 0 \qquad \tilde{F}^\mu = (-B, -E_y, E_x)$ because:

$\tilde{F}^1 = \frac{1}{2}(\epsilon^{102}F_{02} + \epsilon^{120}F_{20}) = \frac{1}{2}(-E_y - E_y) = -E_y$, $\tilde{F}^2 = \frac{1}{2}(\epsilon^{210}F_{10} + \epsilon^{201}F_{01}) = \frac{1}{2}(E_x + E_x) = E_x$, $\tilde{F}^0 = \frac{1}{2}(\overset{1}{\overset{\shortparallel}{\epsilon^{012}}}F_{12} + \overset{-1}{\overset{\shortparallel}{\epsilon^{021}}}F_{21}) = -B$

$\partial_\mu\tilde{F}^\mu = \partial_0(-B) - \partial_1 E_y + \partial_2 E_x = -\partial_t B + \partial_y E_x - \partial_x E_y$ ie. $(\vec{\nabla}\times\vec{E})_z + \partial_t B = 0$.

15) $K\tilde{F}^\nu = -\partial_\mu \cancel{\tfrac{1}{2}}\epsilon^{\mu\nu\rho}\tilde{F}_\rho = -\cancel{\tfrac{1}{2}}\epsilon^{\mu\nu\rho}\partial_\mu\tilde{F}_\rho$

$K^2\tilde{F}^\nu = -\cancel{\tfrac{1}{2}}\epsilon^{\mu\nu\rho}\partial_\mu(K\tilde{F}_\rho) = -\cancel{\tfrac{1}{2}}\underbrace{\epsilon^{\mu\nu\rho}}_{=-\epsilon^{\rho\nu\mu}} \times -\cancel{\tfrac{1}{2}}\underbrace{\epsilon_{\alpha\rho\beta}}_{=\epsilon_{\rho\alpha\beta}}\partial_\mu\partial^\alpha\tilde{F}^\beta = \cancel{\tfrac{1}{4}}(\delta^\nu_\alpha\delta^\mu_\beta - \delta^\nu_\beta\delta^\mu_\alpha)\partial_\mu\partial^\alpha\tilde{F}^\beta$

$= \cancel{\tfrac{1}{4}}(\partial_\beta\partial^\nu\tilde{F}^\beta - \partial_\alpha\partial^\alpha\tilde{F}^\nu) = \partial^\nu\underbrace{(\partial_\beta\tilde{F}^\beta)}_{=0} - \Box\tilde{F}^\nu$

$\to (\Box + K^2)\tilde{F}^\nu = 0 \to \omega = \pm\sqrt{\vec{k}^2 + K^2}$ ie. mass $= |K|$

Ccl

16) topologically massive gauge theory in 2+1

in 3+1, Higgs mechanism but no CS, and Proca is not for gauge field.